**AP Calculus AB Northland Preparatory Academy**

**Pacing Guide SY 2021-2022 Fides Armela Arcos-Rivera**

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| **Timeline and Resources** | **AP-Calculus-Mathematics-Standards** | **Essential Questions (HESS Matrix)/****Learning Goals** | **Vocabulary Content/Academic** |
| **Textbook:**Finney, Demana, Waits, Kennedy and Bressoud.Calculus – Graphical, Numerical, Algebraic. 5th Edition, Pearson Prentice Hall, Boston, MA, 2016Based on AP-Calculus AB/BC – Course and Exam Description at [*https://bit.ly/3bHq6km*](https://bit.ly/3bHq6km)Kahn AcademyDelta Math | **Standards for Mathematical Practices**1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

-*will be applied in all units of study*1. **Implementing Mathematical Processes – Determine expressions and values using mathematical procedures and rules.**

1a. Identify the question to be answered or problem to be solved (not assessed)1b. Identify key and relevant information to answer question or solve a problem (not assessed)1c. Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., Use the chain rule to find the derivative of a composite function)1d. Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems.1e. Apply appropriate mathematical rules or procedures, with and without technology.1f. Explain how approximated value relates to the actual value.1. **Connecting Representations – Translate mathematical information from a single representation or across multiple representations.**

2a. Identify common underlying structures in problems involving different contextual situations.2b. Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.2c. Identify a re-expression of mathematical information presented in a given representation.2d. Identify how mathematical characteristics or properties of functions are related in different representations.2e. Describe the relationships among different representations of functions and derivatives.1. **Justification – Justify reasoning and solutions.**

**3a. Apply technology to develop claims and conjectures (not assessed)**3b. Identify an appropriate mathematical definition, theorem, or test to apply.3c. Confirm whether hypotheses or conditions of a selected definition, theorem or test.3d. Apply an appropriate mathematical definition, theorem, or test.3e. Provide reasons or rationales for solution and conclusions.3f. Explain the meaning or mathematical solutions in context.3g. Confirm that solutions are accurate and appropriate.1. **Communication and Notation – Use correct notation, language, and mathematical conventions to communicate results or solutions.**

4a. Use precise mathematical language.4b. Use appropriate units of measure.4c. Use appropriate mathematical symbols and notation (e.g., represent a derivative using f’(x), y’, dy/dx)4d. Use appropriate graphing techniques.4e. Apply appropriate rounding procedures **BIG IDEA 1: CHANGE (CHA)** Using derivatives to describe rates of change of one variable with respect to another or using definite integrals to describe the net change in one variable over an interval of another allows students to understand change in a variety of contexts. It is critical that students grasp the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus – a central idea in AP Calculus.**BIG IDEA 2: LIMITS (LIM)** Beginning with a discrete model and then considering the consequences of a limiting case allows us to model real-world behavior and to discover and understand important ideas, definitions, formulas, and theorems in Calculus: for example, continuity, differentiation, integration.**BIG IDEA 3: ANALYSIS OF FUNCTIONS (FUN)** Calculus allows us to analyze the behaviors of functions by relating limits to differentiation, integration, and infinite series and relating each of these concepts to the others.  |  |  |
| Unit 0 – Prerequisites for Calculus (10 days) |  | * 1. Linear functions
	2. Function and graphs
	3. Exponential functions
	4. Parametric functions (BC)
	5. Inverse functions
	6. Trigonometric functions
	7. Rational functions
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| Unit 1 – Limits and Continuity (23 days) |  | * 1. Introducing Calculus: Can change occur at an instant?
	2. Defining limits and using limit notation
	3. Estimating limit values from graphs
	4. Estimating limit values from tables
	5. Determining limits using algebraic properties of limits
	6. Determining limits using algebraic manipulation
	7. Selecting procedures for determining limits
	8. Determining limits using squeeze theorem
	9. Connecting multiple representation of limits
	10. Exploring types of 3 discontinuities
	11. Defining continuity at a point
	12. Confirming continuity over an interval
	13. Removing discontinuities
	14. Connecting infinite limits and vertical

 asymptotes* 1. Connecting infinite limits and horizontal

 asymptotes* 1. Working with the Intermediate Value

 Theorem (IVT) | Key Ideas:Average and Instantaneous SpeedDefinition of limitsProperties of limitsOne-sided limits and two-sided limitsSqueezeFinite limit as x >> +/- infinitySqueeze theorem revisitedEnd Behavior Models “Seeing limits as x >> +/- infinityContinuity as a pointContinuous functionAlgebraic combinationsCompositesIntermediate Value Theorem (IVT) for continuous functionsAverage rates of changeTangent to a curveSlope of a curveSpeed revisitedNormal to a curveSpeed revisitedSensitivity |
| Unit 2 – Differentiation: Definition and Basic Derivative Rules (14 days) |  | 2.1 Defining average and instantaneous rates of  change at a point2.2 Defining the derivative of a function and using derivative function2.3 Estimating derivatives of a function at a point2.4 Connecting differentiability and continuity:  determining when derivative do and don’t  exist2.5 Applying the power rule2.6 Derivative Rules: constant, sum, sum  difference and constant multiple.2.7 Derivative of cos x, sin x, e^x and ln(x)2.8 The product rule2.9 The quotient rule | Key Ideas:Definition of derivativeNotationRelationship between the graphs of ***f***and ***f’***Graphing the derivative from dataOne-sided derivativesHow ***f’(a)*** might fail to existDifferentiation implies local linearityNumerical derivatives on a calculatorDifferentiability implies continuityIntermediate Value Theorem for derivativesPositive integerPowers, multiples, sums and differencesProducts and quotientsNegative integer powers of xSecond higher order derivativesInstantameous rates of changeMotion along a lineSensitivity to changeDerivatives in economicsDerivative of the Sine functionDerivative of the Cosine functionSimple harmonic motionJerkDerivative of other basic trigonometric functions |
| Unite 3 – Differentiation: Composite, Implicit and Inverse functions (11 days) |  | 3.1 The chain rule3.2 Implicit differentiation3.3 Differentiating Inverse functions3.4 Differentiating Inverse Trigonometric  Functions3.5 Selecting procedures for calculating  Derivatives3.6 Calculating higher order derivatives | Key Ideas:Derivatives of a composite function; “outside-inside rule”Repeated use of the chain rulePower chain ruleSlopes of parametrized curvesPower chain ruleImplicitly distinct functionLenses, tangents, and normal linesDerivatives of the higher orderRational powers of differentiable functionsDerivatives of inverse functionsDerivatives of arcsineDerivatives arctangentDerivatives of arcsecantDerivatives of the other threeDerivative of e’Derivative of a’Derivative of lnxDerivative of log xPower rule for arbitrary real powers |
| Unit 4 – Contextual Applications of Differentiation (14 days) |  | 4.1 Interpreting the meaning of the derivative in Context 1 CHA4.2 Straight-Line Motion: Connecting Position,  Velocity and Acceleration 1 CHA4.3 Rates of Change in applied contexts other  than Motion4.4 Introduction to Related Rates4.5 Solving Related Rates problems4.6 Approximating Values of a function using local linearity and linearization4.7 Using L’Hospital’s Rule for determining limits of indeterminate form | Key Ideas:Absolute/Global extreme valuesLocal/Relative extreme valuesFinding extreme valuesMean Value TheoremPhysical interpretation of the mean value theoremIncreasing and decreasing functionOrder consequencesFirst derivative test for local extremeConcavity Points of InflectionSecond derivative test for local extremesLearning about functions from derivatovesA strategy for optimizationExamples from business and industryExamples from EconomicsModeling discrete phenomena with differentiable functionsLinear approximationsDifferentialsSensitivity AnalysisAbsolute, relative and percentage of changeNewton’s methodNewton’s method may failRelated rate equationsSolution strategySimulating related motionRelated rate equationsSolutions strategyStimulating related motions |
| Unit 5 – Analytical Applications of Differentiation (16 days) |  | 5.1 Using the Mean Value Theorem5.2 Extreme Value Theorem, Global versus Local  Extrema, Critical Points5.3 Determining Intervals on which a function is increasing and decreasing5.4 Using the First Derivative Test to determine relative (local) extrema5.5 Using the candidates test to determine absolute (global) extrema5.6 Determining the concavity of functions over their domain5.7 Using the second derivative test to determine  Extrema5.8 Sketching graphs of functions and their derivatives5.9 Connecting a function, its first derivative and its second derivative5.10 Introduction to optimization problems5.11 Solving optimization problems5.12 Exploring behaviors of implicit relations |  |
| Unit 6 – Integration and Accumulation of Change (20 days) |  | 6.1 Exploring Accumulation of Change6.2 Approximating Areas with Riemann Sums6.3 Reimann Sums, Summation Notation, and  Definite Integral Notation6.4 The Fundamental Theorem of Calculus and Accumulation Functions6.5 Interpreting the behavior of Accumulation functions involving area6.6 Applying properties of 3 Definite Integrals6.7 The Fundamental Theorem of Calculus and Definite Integrals6.8 Finding Anti-derivatives and Indefinite Integrals: Basic rules and notation6.9 Integrating using substitution6.10 Integrating functions using long division and  completing square6.11 Selecting techniques 1 for anti-differentiation | Key Ideas:Accumulator functionArea under a curveBounded functionCardiac outputCharacteristic function of the rationalsDefinite integralDifferential calculusDummy variablesError boundsFundamental Theorem of CalculusAntiderivativesFundamental Theorem of CalculusEvaluation partIntegral functionIntegral calculusIntegral Evaluation TheoremIntegral of *f* from a to bIntegral signIntegralLower boundLower limit of integrationLRAMMean valueMean Value Theorem for Definite IntegralsMRAMNet areaNINTNorm of PartitionPartitionRectangular Approximation Method (RAM)Regular PartitionReimann SumBRAMSigma NotationSimpson’s RuleSubintervalTotal AreaTrapezoidal RuleUpper boundUpper limit of integrationVariable of integration |
| Unit 7 – Differential Equations (9 days) |  | 7.1 Modeling situations with Differential  Equations 2 FUN7.2 Verifying solutions for differential equations7.3 Sketching slope fields7.4 Reasoning using slope fields7.5 Finding general solutions using separation of  variables7.6 Finding particular solutions using initial  conditions and separation of variables7.7 Exponential models with differential equations | Kay Ideas:Antidifferentiation by partsAntidifferentiation by substitutionCarbon-14 datingCarrying capacityCompounded continuouslyConstant of integrationContinuous interest rateDifferential equationEuler’s methodEvaluate an integralExact differential equationExponential decay constantExponential growth constantFirst-order differential equationsFirst-order linear differential equationGeneral solution to a differential equationGraphical solution of a differential equationHalf-lifeHeavy-side methodIndefinite integralInitial conditionInitial value problemIntegral signIntegrandIntegration by partsLaw of exponential changeLeibniz Notation for integralsLogistic curveLogistic differential equationLogistic growth constantLogistic growth modelNewton’s law of coolingNumerical methodNumerical solution of a differential equationOrder of a differential equationPartial fraction decompositionParticular solutionProper rational functionProperties of indefinite integralsRadioactiveRadioactive decayResistanceProportional to velocitySecond-order of a differential equationSeparation of variablesSlope field Solution to a differential equationSubstitution in definite integralsTabular integrationVariable of integration |
| Unit 8 – Applications of Integration (20 days) |  | 8.1 Finding the average value of a function on an  interval 1 CHA8.2 Connecting position, velocity and acceleration of functions using integrals8.3 Using accumulation functions and definite  integrals in applied contexts8.4 Finding the area between curves expressed as functions of x8.5 Finding the area between curves expressed as functions of y8.6 Finding the area between curves that intersect at more than two points8.7 Volume with cross-sections: Squares and Rectangles8.8 Volumes with cross-sections: Triangles and Semi-circles8.9 Volume with disc method: revolving around other x and y axis8.10 Volume disc method: revolving around other Axes8.11 Volume with washer method8.12 Volume with washer method: revolving  around other axes | Key Ideas:AccumulationArea between curvesCavalier’s TheoremCenter of massConstant force formulaCylindrical shellsDisplacement Fluid forceFluid pressureFoor-poundForce constantGaussian curveHooke’s LawInflation rateJouleMean MomentNet ChangeNewtonNormal curveNormal PDF(Probability Density Function)Solid of revolutionStandard deviationSurface areaTotal distance travelledUniversal gravitational constantVolume by cylindrical shellsVolume by slicingVolume of a solidWeight density work |
| **REVIEW for AP EXAM (24 days)****TEST – May 5th** |  | Review Previous years Free-response Questions |  |
| **Applications of Logistic and Financial Functions (15 days)** |  | Epidemic, Biome carrying capacity, investments loans, car loans, credit cards, car insurance |  |