**AP Calculus AB Northland Preparatory Academy**

**Pacing Guide SY 2021-2022 Fides Armela Arcos-Rivera**

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| **Timeline and Resources** | **AP-Calculus-Mathematics-Standards** | **Essential Questions (HESS Matrix)/**  **Learning Goals** | **Vocabulary Content/Academic** |
| **Textbook:**  Finney, Demana, Waits, Kennedy and Bressoud.  Calculus – Graphical, Numerical, Algebraic. 5th Edition, Pearson Prentice Hall, Boston, MA, 2016  Based on AP-Calculus AB/BC – Course and Exam Description at [*https://bit.ly/3bHq6km*](https://bit.ly/3bHq6km)  Kahn Academy  Delta Math | **Standards for Mathematical Practices**   1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.   -*will be applied in all units of study*   1. **Implementing Mathematical Processes – Determine expressions and values using mathematical procedures and rules.**   1a. Identify the question to be answered or problem to be solved (not assessed)  1b. Identify key and relevant information to answer question or solve a problem (not assessed)  1c. Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., Use the chain rule to find the derivative of a composite function)  1d. Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems.  1e. Apply appropriate mathematical rules or procedures, with and without technology.  1f. Explain how approximated value relates to the actual value.   1. **Connecting Representations – Translate mathematical information from a single representation or across multiple representations.**   2a. Identify common underlying structures in problems involving different contextual situations.  2b. Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.  2c. Identify a re-expression of mathematical information presented in a given representation.  2d. Identify how mathematical characteristics or properties of functions are related in different representations.  2e. Describe the relationships among different representations of functions and derivatives.   1. **Justification – Justify reasoning and solutions.**   **3a. Apply technology to develop claims and conjectures (not assessed)**  3b. Identify an appropriate mathematical definition, theorem, or test to apply.  3c. Confirm whether hypotheses or conditions of a selected definition, theorem or test.  3d. Apply an appropriate mathematical definition, theorem, or test.  3e. Provide reasons or rationales for solution and conclusions.  3f. Explain the meaning or mathematical solutions in context.  3g. Confirm that solutions are accurate and appropriate.   1. **Communication and Notation – Use correct notation, language, and mathematical conventions to communicate results or solutions.**   4a. Use precise mathematical language.  4b. Use appropriate units of measure.  4c. Use appropriate mathematical symbols and notation (e.g., represent a derivative using f’(x), y’, dy/dx)  4d. Use appropriate graphing techniques.  4e. Apply appropriate rounding procedures  **BIG IDEA 1: CHANGE (CHA)** Using derivatives to describe rates of change of one variable with respect to another or using definite integrals to describe the net change in one variable over an interval of another allows students to understand change in a variety of contexts. It is critical that students grasp the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus – a central idea in AP Calculus.  **BIG IDEA 2: LIMITS (LIM)** Beginning with a discrete model and then considering the consequences of a limiting case allows us to model real-world behavior and to discover and understand important ideas, definitions, formulas, and theorems in Calculus: for example, continuity, differentiation, integration.  **BIG IDEA 3: ANALYSIS OF FUNCTIONS (FUN)** Calculus allows us to analyze the behaviors of functions by relating limits to differentiation, integration, and infinite series and relating each of these concepts to the others. |  |  |
| Unit 0 – Prerequisites for Calculus (10 days) |  | * 1. Linear functions   2. Function and graphs   3. Exponential functions   4. Parametric functions (BC)   5. Inverse functions   6. Trigonometric functions   7. Rational functions |  |
| Unit 1 – Limits and Continuity (23 days) |  | * 1. Introducing Calculus: Can change occur at an instant?   2. Defining limits and using limit notation   3. Estimating limit values from graphs   4. Estimating limit values from tables   5. Determining limits using algebraic properties of limits   6. Determining limits using algebraic manipulation   7. Selecting procedures for determining limits   8. Determining limits using squeeze theorem   9. Connecting multiple representation of limits   10. Exploring types of 3 discontinuities   11. Defining continuity at a point   12. Confirming continuity over an interval   13. Removing discontinuities   14. Connecting infinite limits and vertical   asymptotes   * 1. Connecting infinite limits and horizontal   asymptotes   * 1. Working with the Intermediate Value   Theorem (IVT) | Key Ideas:  Average and Instantaneous Speed  Definition of limits  Properties of limits  One-sided limits and two-sided limits  Squeeze  Finite limit as x >> +/- infinity  Squeeze theorem revisited  End Behavior Models “Seeing limits as x >> +/- infinity  Continuity as a point  Continuous function  Algebraic combinations  Composites  Intermediate Value Theorem (IVT) for continuous functions  Average rates of change  Tangent to a curve  Slope of a curve  Speed revisited  Normal to a curve  Speed revisited  Sensitivity |
| Unit 2 – Differentiation: Definition and Basic Derivative Rules (14 days) |  | 2.1 Defining average and instantaneous rates of  change at a point  2.2 Defining the derivative of a function and using  derivative function  2.3 Estimating derivatives of a function at a point  2.4 Connecting differentiability and continuity:  determining when derivative do and don’t  exist  2.5 Applying the power rule  2.6 Derivative Rules: constant, sum, sum  difference and constant multiple.  2.7 Derivative of cos x, sin x, e^x and ln(x)  2.8 The product rule  2.9 The quotient rule | Key Ideas:  Definition of derivative  Notation  Relationship between the graphs of ***f***and ***f’***  Graphing the derivative from data  One-sided derivatives  How ***f’(a)*** might fail to exist  Differentiation implies local linearity  Numerical derivatives on a calculator  Differentiability implies continuity  Intermediate Value Theorem for derivatives  Positive integer  Powers, multiples, sums and differences  Products and quotients  Negative integer powers of x  Second higher order derivatives  Instantameous rates of change  Motion along a line  Sensitivity to change  Derivatives in economics  Derivative of the Sine function  Derivative of the Cosine function  Simple harmonic motion  Jerk  Derivative of other basic trigonometric functions |
| Unite 3 – Differentiation: Composite, Implicit and Inverse functions (11 days) |  | 3.1 The chain rule  3.2 Implicit differentiation  3.3 Differentiating Inverse functions  3.4 Differentiating Inverse Trigonometric  Functions  3.5 Selecting procedures for calculating  Derivatives  3.6 Calculating higher order derivatives | Key Ideas:  Derivatives of a composite function; “outside-inside rule”  Repeated use of the chain rule  Power chain rule  Slopes of parametrized curves  Power chain rule  Implicitly distinct function  Lenses, tangents, and normal lines  Derivatives of the higher order  Rational powers of differentiable functions  Derivatives of inverse functions  Derivatives of arcsine  Derivatives arctangent  Derivatives of arcsecant  Derivatives of the other three  Derivative of e’  Derivative of a’  Derivative of lnx  Derivative of log x  Power rule for arbitrary real powers |
| Unit 4 – Contextual Applications of Differentiation (14 days) |  | 4.1 Interpreting the meaning of the derivative in  Context 1 CHA  4.2 Straight-Line Motion: Connecting Position,  Velocity and Acceleration 1 CHA  4.3 Rates of Change in applied contexts other  than Motion  4.4 Introduction to Related Rates  4.5 Solving Related Rates problems  4.6 Approximating Values of a function using  local linearity and linearization  4.7 Using L’Hospital’s Rule for determining limits  of indeterminate form | Key Ideas:  Absolute/Global extreme values  Local/Relative extreme values  Finding extreme values  Mean Value Theorem  Physical interpretation of the mean value theorem  Increasing and decreasing function  Order consequences  First derivative test for local extreme  Concavity  Points of Inflection  Second derivative test for local extremes  Learning about functions from derivatoves  A strategy for optimization  Examples from business and industry  Examples from Economics  Modeling discrete phenomena with differentiable functions  Linear approximations  Differentials  Sensitivity Analysis  Absolute, relative and percentage of change  Newton’s method  Newton’s method may fail  Related rate equations  Solution strategy  Simulating related motion  Related rate equations  Solutions strategy  Stimulating related motions |
| Unit 5 – Analytical Applications of Differentiation (16 days) |  | 5.1 Using the Mean Value Theorem  5.2 Extreme Value Theorem, Global versus Local  Extrema, Critical Points  5.3 Determining Intervals on which a function is  increasing and decreasing  5.4 Using the First Derivative Test to determine  relative (local) extrema  5.5 Using the candidates test to determine absolute  (global) extrema  5.6 Determining the concavity of functions over  their domain  5.7 Using the second derivative test to determine  Extrema  5.8 Sketching graphs of functions and their  derivatives  5.9 Connecting a function, its first derivative and  its second derivative  5.10 Introduction to optimization problems  5.11 Solving optimization problems  5.12 Exploring behaviors of implicit relations |  |
| Unit 6 – Integration and Accumulation of Change (20 days) |  | 6.1 Exploring Accumulation of Change  6.2 Approximating Areas with Riemann Sums  6.3 Reimann Sums, Summation Notation, and  Definite Integral Notation  6.4 The Fundamental Theorem of Calculus and  Accumulation Functions  6.5 Interpreting the behavior of Accumulation  functions involving area  6.6 Applying properties of 3 Definite Integrals  6.7 The Fundamental Theorem of Calculus and  Definite Integrals  6.8 Finding Anti-derivatives and Indefinite  Integrals: Basic rules and notation  6.9 Integrating using substitution  6.10 Integrating functions using long division and  completing square  6.11 Selecting techniques 1 for anti-differentiation | Key Ideas:  Accumulator function  Area under a curve  Bounded function  Cardiac output  Characteristic function of the rationals  Definite integral  Differential calculus  Dummy variables  Error bounds  Fundamental Theorem of Calculus  Antiderivatives  Fundamental Theorem of Calculus  Evaluation part  Integral function  Integral calculus  Integral Evaluation Theorem  Integral of *f* from a to b  Integral sign  Integral  Lower bound  Lower limit of integration  LRAM  Mean value  Mean Value Theorem for Definite Integrals  MRAM  Net area  NINT  Norm of Partition  Partition  Rectangular Approximation Method (RAM)  Regular Partition  Reimann Sum  BRAM  Sigma Notation  Simpson’s Rule  Subinterval  Total Area  Trapezoidal Rule  Upper bound  Upper limit of integration  Variable of integration |
| Unit 7 – Differential Equations (9 days) |  | 7.1 Modeling situations with Differential  Equations 2 FUN  7.2 Verifying solutions for differential equations  7.3 Sketching slope fields  7.4 Reasoning using slope fields  7.5 Finding general solutions using separation of  variables  7.6 Finding particular solutions using initial  conditions and separation of variables  7.7 Exponential models with differential equations | Kay Ideas:  Antidifferentiation by parts  Antidifferentiation by substitution  Carbon-14 dating  Carrying capacity  Compounded continuously  Constant of integration  Continuous interest rate  Differential equation  Euler’s method  Evaluate an integral  Exact differential equation  Exponential decay constant  Exponential growth constant  First-order differential equations  First-order linear differential equation  General solution to a differential equation  Graphical solution of a differential equation  Half-life  Heavy-side method  Indefinite integral  Initial condition  Initial value problem  Integral sign  Integrand  Integration by parts  Law of exponential change  Leibniz Notation for integrals  Logistic curve  Logistic differential equation  Logistic growth constant  Logistic growth model  Newton’s law of cooling  Numerical method  Numerical solution of a differential equation  Order of a differential equation  Partial fraction decomposition  Particular solution  Proper rational function  Properties of indefinite integrals  Radioactive  Radioactive decay  Resistance  Proportional to velocity  Second-order of a differential equation  Separation of variables  Slope field  Solution to a differential equation  Substitution in definite integrals  Tabular integration  Variable of integration |
| Unit 8 – Applications of Integration (20 days) |  | 8.1 Finding the average value of a function on an  interval 1 CHA  8.2 Connecting position, velocity and acceleration  of functions using integrals  8.3 Using accumulation functions and definite  integrals in applied contexts  8.4 Finding the area between curves expressed as  functions of x  8.5 Finding the area between curves expressed as  functions of y  8.6 Finding the area between curves that intersect  at more than two points  8.7 Volume with cross-sections: Squares and  Rectangles  8.8 Volumes with cross-sections: Triangles and  Semi-circles  8.9 Volume with disc method: revolving around  other x and y axis  8.10 Volume disc method: revolving around other  Axes  8.11 Volume with washer method  8.12 Volume with washer method: revolving  around other axes | Key Ideas:  Accumulation  Area between curves  Cavalier’s Theorem  Center of mass  Constant force formula  Cylindrical shells  Displacement  Fluid force  Fluid pressure  Foor-pound  Force constant  Gaussian curve  Hooke’s Law  Inflation rate  Joule  Mean  Moment  Net Change  Newton  Normal curve  Normal PDF  (Probability Density Function)  Solid of revolution  Standard deviation  Surface area  Total distance travelled  Universal gravitational constant  Volume by cylindrical shells  Volume by slicing  Volume of a solid  Weight density work |
| **REVIEW for AP EXAM (24 days)**  **TEST – May 5th** |  | Review Previous years Free-response Questions |  |
| **Applications of Logistic and Financial Functions (15 days)** |  | Epidemic, Biome carrying capacity, investments loans, car loans, credit cards, car insurance |  |