AP® Calculus BC: Sample Syllabus 1 Contents

Curricular Requirements

CR1a   The course is structured around the enduring understandings within Big Idea 1: Limits.
- See page 2

CR1b   The course is structured around the enduring understandings within Big Idea 2: Derivatives.
- See pages 2, 3

CR1c   The course is structured around the enduring understandings within Big Idea 3: Integrals and the Fundamental Theorem of Calculus.
- See page 3

CR1d   The course is structured around the enduring understandings within Big Idea 4: Series.
- See page 5

CR2a   The course provides opportunities for students to reason with definitions and theorems.
- See page 3

CR2b   The course provides opportunities for students to connect concepts and processes.
- See page 3

CR2c   The course provides opportunities for students to implement algebraic/computational processes.
- See page 2

CR2d   The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them.
- See page 2

CR2e   The course provides opportunities for students to build notational fluency.
- See page 4

CR2f   The course provides opportunities for students to communicate mathematical ideas in words, both orally and in writing.
- See page 2

CR3a   Students have access to graphing calculators.
- See page 1

CR3b   Students have opportunities to use calculators to solve problems.
- See page 4

CR3c   Students have opportunities to use a graphing calculator to explore and interpret calculus concepts.
- See page 2

CR4   Students and teachers have access to a college-level calculus textbook.
- See page 1

AP Calculus BC Syllabus

Course Overview:

This is a college-level calculus course designed to meet the Advanced Placement curricular requirements for Calculus BC (equivalent to one year of college calculus). The major topics of this course are limits, derivatives, integrals, the Fundamental Theorem of Calculus, and series. We will investigate and analyze course topics using equations, graphs, tables, and words, with a particular emphasis on a conceptual understanding of calculus. Applications, in particular to solid geometry and physics, will be studied where appropriate.

Technology Requirement:

You will need a handheld graphing calculator every day. Both TI-84 and TI-Nspire CAS will be used with the class projector; any AP-approved calculator will work. Your teacher has TI-84 calculators available for checkout, if desired. You are encouraged to use your own device, as appropriate, for class work and projects. **[CR3a]**

> **[CR3a]** — Students have access to graphing calculators.

Classroom Expectations:

Every student is an important member of this class. You are expected to actively participate, stay engaged, and discuss ideas. The classroom is arranged in teams of four so that you may collaborate and share ideas freely. Most classes will include some group work, ranging from a warm-up (often using a past AP exam question) to an exploration. You will use the individual and group-sized whiteboards to aid in this sharing. Groups will be asked to present their work to the class on a rotating basis.

Everyone is encouraged to form a study group for additional support. Your teacher will be available before (even before zero hour) and after school, as posted, for extra help. Calculus is a challenging, rigorous course. We can do this together.

Textbook:

Finney, Demana, Waits, and Kennedy. *Calculus: Graphical, Numerical, Algebraic*. 3rd ed. Boston: Pearson Prentice Hall, 2003 . **[CR4]**

> **[CR4]** — Students and teachers have access to a college-level calculus textbook.

Additional Resources:

The class   site contains PDF files of all notes and handouts. There are also sections with links to online applications, study guides, videos, and AP review resources.

Course Outline

Limits and Continuity (Chapter 2) [CR1a]:

- 2.1 Rates of Change and Limits
- 2.2 Limits Involving Infinity
- 2.3 Continuity
- 2.4 Rates of Change and Tangent Lines

    [CR1a] — The course is structured around the enduring understandings within Big Idea 1: Limits.

Sample Activities:

Points of (dis)continuity: Students explore limits at discontinuities in four ways: first, using the table feature on their calculators with decreasing increments; second, using algebraic techniques to "simplify" the expressions given as formulas; third, using the graph trace feature on their calculators; and fourth, using descriptions of functions written in words to create graphs that match the verbal descriptions. Student work for the activity includes a written summary, using complete sentences, of their findings that compares and contrasts jump, removable, and asymptotic discontinuities. They give an oral presentation describing how the different representations reveal the discontinuities in different ways. **[CR2c] [CR2d] [CR2f] [CR3c]**

Limit War: This game uses a set of 24 cards with limits expressed analytically. For each round, players find the limits on their card and announce the answer to their group. The player with the highest result (i.e., infinity, finite values in descending order, negative infinity, and does not exist) wins the cards. Decks are created so that each student receives an equal number of cards with special values in a three-person game. The student with the most cards at the end of the allotted time wins. The game is an introductory week activity with calculators allowed and is repeated as a test review with no calculators. In the cut-throat version, players can steal if their opponent incorrectly evaluates their limit.

    [CR2c] — The course provides opportunities for students to implement algebraic/computational processes.

    [CR2d] — The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them.

    [CR2f] — The course provides opportunities for students to communicate mathematical ideas in words, both orally and in writing.

    [CR3c] — Students have opportunities to use a graphing calculator to explore and interpret calculus concepts.

Derivatives (Chapter 3) [CR1b]:

- 3.1 Derivative of a Function
- 3.2 Differentiability
- 3.3 Rules for Differentiation
- 3.4 Velocity and Other Rates of Change
- 3.5 Derivatives of Trigonometric Functions
- 3.6 Chain Rule
- 3.7 Implicit Differentiation
- 3.8 Derivatives of Inverse Trigonometric Functions
- 3.9 Derivatives of Exponential and Logarithmic Functions

    [CR1b] — The course is structured around the enduring understandings within Big Idea 2: Derivatives.

Sample Activities:

Derivative at a Point, Act 1: Students use a worksheet to explore the derivative function using the limit definition of the derivative at a point. For several points, students compute the difference quotient algebraically using the definition and use a table and/or graph on their calculators to evaluate the limits and interpret their results in terms of the definition to decide if the derivative does or does not exist at each point. The original function and its derivative values are plotted on two graphs with the same horizontal axes. Students trade papers and validate each other's answers. **[CR2a]**

Derivative at a Point, Act 2: Students explore the derivative function using tangent lines at a point. Students are given a graph of some function f on easel-size grid paper. At several points, students draw tangent lines and use their slopes to estimate the derivative. Students plot the derivative values on the same set of axes. Graphs may include discontinuous and non-differentiable points. Results are presented to the class.

Derivative at a Point, Act 3: Students use Desmos to create a slider showing a function and its derivative. The graph shows the tangent line sliding along the curve as its slope values are plotted. Polynomial, rational, exponential, logarithmic, trigonometric, and inverse trigonometric functions are used as the parent function on different days.

> **[CR2a]** — The course provides opportunities for students to reason with definitions and theorems.

Applications of Derivatives (Chapter 4):

- 4.1 Extreme Values of Functions
- 4.2 Mean Value Theorem **[CR1b: Mean Value Theorem]**
- 4.3 Connecting $f'(x)$ and $f''(x)$ and with the Graph of $f(x)$
- 4.4 Modeling and Optimization
- 4.6 Related Rates

> **[CR1b]** — The course is structured around the enduring understandings within Big Idea 2: Derivatives.

Sample Activities:

Search for f: Using the graph of the derivative $f'(x)$, students determine key features of $f(x)$ (such as increasing/decreasing intervals, local extrema, points of inflection, and concavity intervals) and create a graph of $f(x)$. They exchange $f(x)$ graphs with a different group and make a graph of $f'(x)$ to match their new $f(x)$. The groups meet together to check $f'(x)$ graphs and resolve any differences. **[CR2b]**

Rate the Rates: Students create short videos to act out and solve a related rates or optimization problem. Videos show the original problem set up and have a pause point opportunity that may be used to have the class solve the problem.

Curve Artists: Pairs of students sit so that one is facing the SMART Board and the other has his/her back to it. The student looking at the board uses sentences involving first derivatives to describe a displayed graph. The other student uses these directions to produce a graph on a small whiteboard without showing their work to their partner. Students reverse roles and repeat for a new graph.

> **[CR2b]** — The course provides opportunities for students to connect concepts and processes.

Integrals and the Fundamental Theorem of Calculus – Parts I and II (Chapter 5) [CR1c]:

- 5.1 Estimating with Finite Sums

- 5.5 Trapezoidal Rule
- 5.2 Definite Integrals
- 5.3 Definite Integrals and Antiderivatives
- 5.4 Fundamental Theorem of Calculus – Parts I and II

[CR1c] — The course is structured around the enduring understandings within Big Idea 3: Integrals and the Fundamental Theorem of Calculus.

Sample Activities:

Riemann Sums to Definite Integrals: Students write an expression for an approximation of the area between the horizontal axis and the graph of $f(x)$ for a particular function given as a formula on a specified interval as a left, right, and midpoint Riemann sum using n subdivisions. They then use a Desmos graph with slider to explore sums. The file superimposes rectangular areas on the graph of $f(x)$, showing the sum value. The software allows for left, right, and midpoint sums. The slider increases the number of partitions to explore precision. Finally, students write limits of their Riemann suns as n goes to infinity, then identify each as a definite integral, and use the Fundamental Theorem of Calculus to evaluate the integral. **[CR2e]**

[CR2e] — The course provides opportunities for students to build notational fluency.

More with Integrals (Chapter 6):

- 6.0 Anti-derivatives
- 6.2 Antidifferentiation by Substitution
- 6.3 Antidifferentiation by Parts
- 6.5 Logistic Growth (Antidifferentiation by Partial Fractions)

Sample Activities:

Picking Parts: Students work in groups to start a worksheet of several integration by parts exercises. The goal is to determine how to choose the function for u. For each problem, groups write out the first step of (at least) two potential by parts solutions choosing a different function for u each time. At the end of the activity, groups are given magnets for the five basic function types (i.e., logarithmic, inverse trigonometric, algebraic, trigonometric, and exponential) and asked to put them in descending order for the best choice (in general) for u and to justify their order.

Applications of Definite Integrals (Chapter 7):

- 7.1 Integral as Net Change
- 7.2 Areas in the Plane
- 7.3 Volumes

Sample Activities:

Honeycomb Volume: In pairs, students find the volume of paper party decorations. Students trace the decoration, measure the outline to construct a data table, and use regression to determine the curve(s) of best fit for their object. They write integrals to represent the volume of their objects as solids of revolution and compute the volumes on their calculators. Pairs will assess the reasonableness of answers and compare results with another team. **[CR3b]**
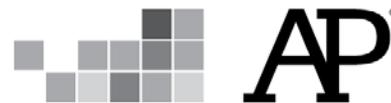
Solids of Revolution: Students will bring a small object from home that is a solid of revolution. Students measure their objects to create a graph whose rotation about the x-axis would produce their object. Then, they use regression and multiple integrals to compute their object's theoretical volume. Students then use displacement to determine the actual volume and calculate their percentage of error.

> **[CR3b]** — Students have opportunities to use calculators to solve problems.

Improper Integrals (Chapter 8):

- 8.2 L'Hospital's Rule
- 8.3 Relative Rates of Growth
- 8.4 Improper Integrals
- 7.5 Applications from Science and Statistics

Sample Activities:

Limit War Revisited: An expansion set of indeterminate form cards is added to the deck. Students work in pairs to examine limits in the context of graphical and analytical representations of rational functions to determine if and when L'Hospital's Rule applies. They share their answers with a larger group and jointly resolve any differences.

Series, Power Series, and Taylor Polynomials (Chapter 9) [CR1d]:

- 9.1 Power Series
- 9.2 Taylor Polynomials
- 9.3 Taylor's Theorem
- 9.4 Radius of Convergence
- 9.5 Testing Convergence at Endpoints

> **[CR1d]** — The course is structured around the enduring understandings within Big Idea 4: Series.

Sample Activities:

I Have, Who Has: A set of "I have _____, who has _____?" was developed to use with basic infinite series. Students work in pairs to determine if one of their cards is the answer to the posed problem. For example, "Who has a geometric series that sums to 6?" matches with "I have $\sum_{k=0}^{\infty}\frac{3}{2^k}$."

Modifying Maclaurin: Each group is given a common Maclaurin series, $f(x)$, on a colored notecard #1 with the first four nonzero terms and general term listed. They are asked to modify the series [for example, $f(4x)$ or $x^2 f(x)$], write the new series on the same-colored notecard #2, and add a new modification for the next group to perform. After four modifications are completed, the modified series is given to a new group with the challenge to decode the modification. The notecards are put in order to check.

Calculus with Parametrics, Vectors, and Polars (Chapter 10):

- 10.1 Parametric Functions
- 10.2 Vectors in the Plane
- 10.3 Polar Functions

Sample Activities:

Polar Art: Students use two to four polar curves to create a design on Desmos, print their graph, and color it like a stained glass window. Students solve for the amount of each color of glass (area) and the amount of copper wire (boundary length) needed to produce their window.

Differential Equations (Chapter 6):

- 6.1 Slope Fields and Euler's Method
- 6.4 Exponential Growth and Decay
- 6.5 Logistic Growth

Sample Activities:

Slope Field Card Sort: Sets of cards include 10 differential equations represented symbolically, as a slope field, and by a verbal description. Students match the cards to bring together all three representations.

Slope Field Walk: On an easel-sized piece of graph paper, students produce a slope field for a differential equation. A general solution is sketched using an initial value. The next day, students produce a linear walk through their slope field using steps of a given size to draw out Euler's method. They then repeat the activity with smaller steps. Students revisit their differential equations the following week and solve them symbolically using separation of variables. Students then solve the initial value problems and compare the actual solution curves with their initial sketches.

Newton's Law: Using temperature probes and CBL's, students work with Newton's law of cooling. Groups use containers with varying insulation levels and either hot or cold beverages to determine which container is best.

AP Review:

Review for the AP test is ongoing throughout the year. Past multiple choice and free response problems are regularly used for in class warm-ups. Problem sets centered on particular themes and written in AP style are used at least twice in each chapter. Students work through at least four complete practice AP exams (i.e., winter break, spring break, first week of April, and last week of April), with an optional Sunday afternoon full practice exam.

During the two- to three-week review period, groups present major topics of the course and students work through an AP practice book. Past AP Practice tests are analyzed using a self-assessment rubric to determine individual areas for deeper study. This leads to personal review projects.